SELFSIMILAR MOTIONS WITH HYDRAULIC WAVES IN SHALLOW WATER

(AVTOMODEL'NYE DVIZHENIE S GIDRAVLICHESKIMI VOLNAMI V SLUCHAE "MELKOI VODY")

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The analogy is well known between the motion of a compressible gas and the motion of a liquid with a free surface in a gravity field, when the depth of the liquid layer is small in comparison with the characteristic dimensions of the problem ("shallow water", see, for instance, reference [1]). It allows us to use the "shallow water" case to solve one-dimensional problems of transient gas motion [2,3]. Thus, if we introduce the quantities

$$\rho = \rho_f h, \qquad p = \frac{1}{2}\rho_f g h^2 \tag{1}$$

where ρ_f is the liquid density, ρ and p, density and pressure of some fictitious gas with ratio of specific heats equal to $\kappa = 2.0$, then the equations of motion of the liquid coincide with those of the adiabatic flow of this fictitious gas. However, the conditions for the hydraulic wave (jump h) differ from those of a shock wave in a gas, a circumstance which intoduces special problems.

In a similar manner [4] we will study selfsimilar motions in shallow water with hydraulic waves propagating at constant velocity D over a liquid of depth h_1 at rest. The parameters following the hydraulic wave are determined by the expressions

$$\frac{h_2}{h_1} = \sqrt{\frac{1}{4} + 2\frac{c^2}{a_1^2}} - \frac{1}{2} = \Phi\left(\frac{D}{a_1}\right),$$

$$a_1 = \sqrt{gh_1} \tag{2}$$

$$v_2 = D\left(1 - \frac{1}{\Phi}\right), \qquad \rho_2 = \rho_1\Phi, \qquad p_2 = p_1\Phi^2$$

where a_1 is the velocity of propagation of small disturbances, v_2 is the velocity of liquid motion directly behind the hydraulic wave.

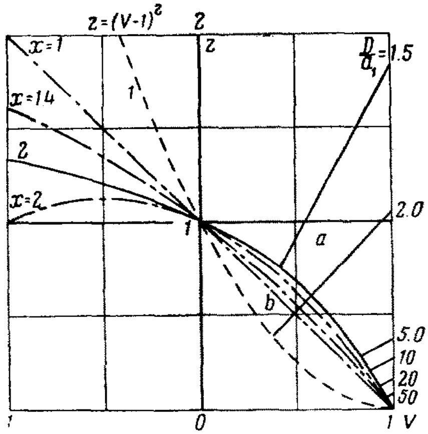

Fig. 1.

The plane hydraulic wave front corresponds to a trivial flow at constant velocity behind the wave $v = v_2 = \text{const}$. It is interesting to study similarity to radial flows with cylindrical hydraulic wave front, which could form the model for corresponding tectonic processes under the influence of the water surface. The similarity solution will be a function of the dimensionless combination $\lambda = \beta r / tD$ and can be determined by integrating the following system of equations [2]:

$$\frac{dz}{dV} = \frac{z}{V}\,\frac{(V-1)(3V-2)-2z}{(V-1)^2-2z}, \qquad \frac{d\ln\lambda}{dV} = \frac{z-(V-1)^2}{V\left[(V-1)^2-2z\right]}, \qquad R = \varkappa z \lambda^2 \quad (3)$$

$$v = \frac{r}{t}\,V(\lambda), \qquad \rho = \rho_1 R(\lambda), \qquad p = \frac{\rho_1 r^2}{t^2}\,P(\lambda), \qquad z = \frac{2P}{R}$$

Consistent with (2) the boundary conditions behind the hydraulic wave can be written thus

$$V_2 = \frac{\Phi - 1}{\Phi}, \qquad z_2 = \Phi\,\frac{a_1^2}{D^2}, \qquad R_2 = \Phi \quad (4)$$

The relation $V = f(z_2)$ is shown in Fig. 1, where curves are also displayed

$$z_2 = (1 - V_2)\left(1 + \frac{\varkappa - 1}{2} V_2\right)$$

for gases with $\kappa = 2$, 1.4 and 1.0.

On the line $V_2 = f(z_2)$ the integral curves of equation (3) start, examples of integral curves being shown in Fig. 1. There are four kinds of flow which correspond to the type under discussion [4], of which for $V_2 > 0$ there exist two kinds of flow in which the liquid comes to rest in front of the approaching hydraulic wave.

(a) Flow with diverging or separating hydraulic wave for $t > 0$

(b) Flow with converging hydraulic wave for $t < 0$.

Integral curves for flow (a) anu at the line $V = 1$ which corresponds to a cylindrical piston expanding radially at constant velocity. The flow (b) is limited to the plane Vz by parabola $z = (V - 1)^2$ (see references [2,4]).

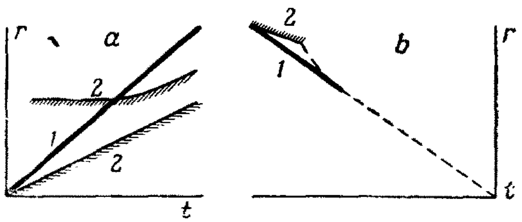

Fig. 2.

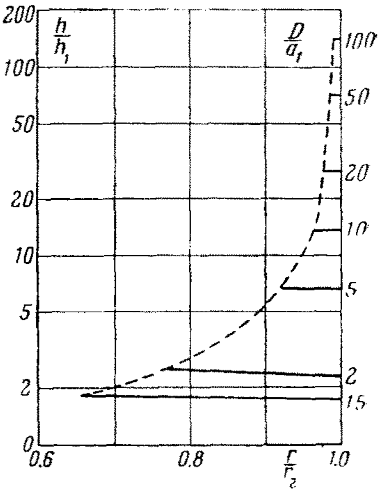

Fig. 3.

Figure 2 illustrates flows in the physical plane for the cases under discussion, (1 is the hydraulic wave, 2 the piston). Figure 3 shows the distribution of liquid depth behind a diverging hydraulic wave, and Fig.4 is the same but for a converging wave.
Change in relative energy

$$E^\circ = \frac{E}{\rho_{\text{ж}} g h_1^2 \pi r_2^2}$$

expended by the piston as a function of D/a_1 (for the flow (a)) is illustrated in Fig. 5.

Other well known transient motions, such as the case of a point explosion against backpressure [2] can also be dealt with by similarity methods in this manner.

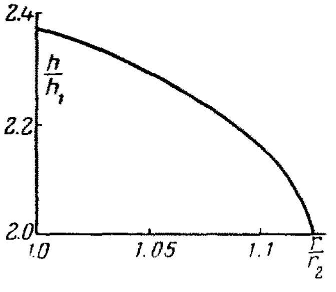

Fig. 4.

With a very powerful hydraulic wave $(D/a_1) \to \infty$ the ratio h_2/h_1 tends to infinity; therefore, to flows of the type of a powerful explosion in gas [2], in the case of "shallow water" there corresponds the outflow [5] of a constant mass of liquid for $h_1 = 0$. As in the case of gas, in addition to the powerful point explosion [2] there corresponds a similar motion which corresponds to a peripheral explosion [6]. For the "shallow water" case with a cylindrical front [6] the solution corresponds to the following distribution of depth

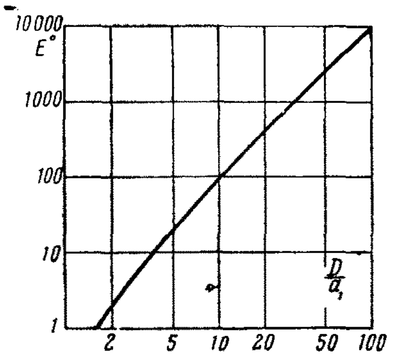

Fig. 5.

$$h = \frac{r^2}{8gt^2} \quad \text{for } t < 0 \tag{5}$$

bounded by cylindrical surfaces

$$r_{1,2} \sim \sqrt{t_{1,2}} \tag{6}$$

This type of peripheral "ring" flow extends continuously towards the centre as depth increases.

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